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*Original Citation:*

*Availability:*

This version is available <http://hdl.handle.net/2318/1589128> since 2016-08-17T14:11:15Z

*Published version:*

DOI:10.1016/j.nuclphysbps.2015.09.212

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# The Inclusive Determination of $|V_{cb}|$

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## Abstract

We present preliminary updated global fits to the moments of semileptonic  $B$  decay distributions and extract  $|V_{cb}|$  in addition to the heavy quark masses ( $m_b, m_c$ ), and non-perturbative heavy quark expansion parameters ( $\mu_\pi, \mu_G, \rho_D, \rho_{LS}$ ). Included are both NNLO perturbative corrections and presently calculated perturbative corrections to the power suppressed operators. We also discuss briefly the method of integration used to compute the  $O(\alpha_s \Lambda_{\text{QCD}}^2/m_b^2)$  corrections.

**Keywords:** CKM,  $V_{cb}$ , Inclusive, Fit,

## 1. Introduction

A precise theoretical determination of the CKM matrix element  $V_{cb}$  is imperative for an accurate exploration of heavy flavor phenomena. The  $b \rightarrow c$  transition is important for the analysis of CP violation in the Standard Model(SM), and in constraining both flavor violating processes and the CKM unitarity triangle values of  $\bar{\rho}$  and  $\bar{\eta}$ .

The determination of  $V_{cb}$  from inclusive semileptonic  $B$  decays is based on an Operator Product Expansion (OPE) that allows us to express the widths and the moments of the kinematic distributions of  $B \rightarrow X_{u,c} \ell \nu$  as double expansions in  $\alpha_s$  and  $\Lambda_{\text{QCD}}/m_b$ . These corrections are now known perturbatively to  $O(\alpha_s^2)[1, 2, 3, 4]$ ,  $O(\alpha_s \Lambda_{\text{QCD}}^2/m_b^2)[5, 6, 7]$ , and to  $O(\Lambda_{\text{QCD}}^3/m_b^3)[8, 9]$  in the Heavy Quark Expansion (HQE)[10].

$$\begin{aligned}
M_i &= M_i^{(0)} + M_i^{(\pi,0)} \frac{\mu_\pi^2}{m_b^2} + M_i^{(G,0)} \frac{\mu_G^2}{m_b^2} \\
&+ M_i^{(D,0)} \frac{\rho_D^3}{m_b^3} + M_i^{(LS,0)} \frac{\rho_{LS}^3}{m_b^3} \\
&+ \left( \frac{\alpha_s}{\pi} \right) \left[ M_i^{(1)} + M_i^{(\pi,1)} \frac{\mu_\pi^2}{m_b^2} + M_i^{(G,1)} \frac{\mu_G^2}{m_b^2} \right. \\
&\quad \left. + M_i^{(D,1)} \frac{\rho_D^3}{m_b^3} + M_i^{(LS,1)} \frac{\rho_{LS}^3}{m_b^3} \right] \\
&+ \left( \frac{\alpha_s}{\pi} \right)^2 M_i^{(2)} + O(m_b^{4,5}, \alpha_s^3).
\end{aligned} \tag{1}$$

In Equation 1 one can see the different elements of the double expansion for a given observable. For the observables that concern us, the green terms have been calculated previously, the blue terms are the new corrections that have been added to the calculations of the necessary observables, and the red terms are currently being calculated or have been approximated in various methods[11]. Each observable is dependent on the masses of the heavy quarks,  $m_b$  and  $m_c$ ,  $\alpha_s$  the strong coupling constant, and the matrix elements of local operators operating on the  $B$ -meson at increasing powers of  $1/m_b$ . To the order currently calculated this includes  $\mu_\pi^2$  and  $\mu_G^2$  at  $O(1/m_b^2)$ ,  $\rho_D^3$  and  $\rho_{LS}^3$  at  $O(1/m_b^3)$ . These matrix elements can be constrained by various measurements of the first 3 central moments of the lepton energy and hadronic mass distributions of  $B \rightarrow X_c \ell \nu$ . Each observable is measured varying the low-energy cut-off of the leptonic spectrum and these have been measured to good accuracy at the  $B$ -factories, CLEO, DELPHI and CDF.

Combining with the total semileptonic width, these parameters can then be used to extract  $|V_{cb}|$ . In the past this strategy has been quite successful and has allowed for a  $\sim 2\%$  determination of  $V_{cb}$  from inclusive decays [12]. Additional motivation for increasing the precision of the extracted parameters is a desire to resolve a  $\sim 2\sigma$  discrepancy that exists between the current inclusive

determination and the most precise  $|V_{cb}|$  determination from the exclusive  $B \rightarrow D^* \ell \nu$  at zero recoil with a lattice calculation of the form-factor [12, 13]. It should be noted however that the zero-recoil form-factor estimate based on heavy quark sum rules leads to  $|V_{cb}|$  in good agreement with the inclusive result [14].

## 2. Theoretical Error and Correlations

The previous procedures used in the semileptonic fits [15, 16, 17] have been recently re-examined and a few relevant issues are worthy of ones concern : *i*) the theoretical uncertainties and how they are implemented in the fit, and *ii*) the inclusion of additional constraints on the parameters. For a thorough overview we direct the reader to consult [18]. In brief, considering 100% correlation between an observable and the same observable with a different cut on the leptonic energy is too strong of a stipulation. This requirement is relaxed and tested with various parametrizations of the correlations between observables.

In these proceedings we will be using a limited selection of the theoretical correlation options and additional mass constraints. The full results will be published in a following submission. We choose to look solely at the scenario that provided the most accurate and reliable results at  $\mathcal{O}(\alpha_s^2)$ , which is the case of a functional theoretical correlation between moments [D], and using the constraints of  $m_c = \bar{m}_c(3\text{GeV})$ , and not including external  $m_b$  constraints. This is performed with  $\alpha_s(\mu) = \alpha_s(m_b) = 0.22$ , and  $\mu_G^2(\mu_{\mu G}) = \mu_G^2(m_b)$ .

## 3. Experimental Observables

The relevant quantities used in the fit are the first 3 moments of the leptonic energy spectrum as a function of a cut on the lower energy limit,

$$\langle E_\ell^n \rangle_{E_\ell > E_{cut}} = \frac{\int_{E_{cut}}^{E_{max}} dE_\ell E_\ell^n \frac{d\Gamma}{dE_\ell}}{\int_{E_{cut}}^{E_{max}} dE_\ell \frac{d\Gamma}{dE_\ell}}, \quad (2)$$

which are measured for  $n$  up to 4, as well as the ratio  $R^*$  between the rate with and without a cut

$$R^*(E_{cut}) = \frac{\int_{E_{cut}}^{E_{max}} dE_\ell \frac{d\Gamma}{dE_\ell}}{\int_0^{E_{max}} dE_\ell \frac{d\Gamma}{dE_\ell}}. \quad (3)$$

$R^*$  is needed to relate the actual measurement of the rate to one with a cut, from which one can then extract  $|V_{cb}|$ .

Due to the high degree of correlation in the first three linear moments, it is beneficial to instead study the central moments, including the variance and asymmetry of

the lepton energy distribution. In our procedure we will consider only  $R^*$  and the first 3 central moments,

$$\begin{aligned} \ell_1(E_{cut}) &= \langle E_\ell \rangle_{E_\ell > E_{cut}}, \\ \ell_{2,3}(E_{cut}) &= \langle (E_\ell - \langle E_\ell \rangle)^{2,3} \rangle_{E_\ell > E_{cut}}. \end{aligned} \quad (4)$$

In the case of the moments of the hadronic invariant mass distribution we similarly consider the central moments

$$\begin{aligned} h_1(E_{cut}) &= \langle M_X^2 \rangle_{E_\ell > E_{cut}}, \\ h_{2,3}(E_{cut}) &= \langle (M_X^2 - \langle M_X^2 \rangle)^{2,3} \rangle_{E_\ell > E_{cut}}. \end{aligned} \quad (5)$$

One will find below in Table 1 a list of the experimental data used in our fit of the HEQ parameters and heavy quark masses used in extracting  $V_{cb}$ .

	experiment	values of $E_{cut}(\text{GeV})$	Ref.
$R^*$	BaBar	0.6, 1.2, 1.5	[19, 20]
$\ell_1$	BaBar	0.6, 0.8, 1, 1.2, 1.5	[19, 20]
$\ell_2$	BaBar	0.6, 1, 1.5	[19, 20]
$\ell_3$	BaBar	0.8, 1.2	[19, 20]
$h_1$	BaBar	0.9, 1.1, 1.3, 1.5	[19]
$h_2$	BaBar	0.8, 1, 1.2, 1.4	[19]
$h_3$	BaBar	0.9, 1.3	[19]
$R^*$	Belle	0.6, 1.4	[21]
$\ell_1$	Belle	1, 1.4	[21]
$\ell_2$	Belle	0.6, 1.4	[21]
$\ell_3$	Belle	0.8, 1.2	[21]
$h_1$	Belle	0.7, 1.1, 1.3, 1.5	[22]
$h_2$	Belle	0.7, 0.9, 1.3	[22]
$h_{1,2}$	CDF	0.7	[23]
$h_{1,2}$	CLEO	1, 1.5	[24]
$\ell_{1,2,3}$	DELPHI	0	[25]
$h_{1,2,3}$	DELPHI	0	[25]

Table 1: Experimental data used in the fits unless otherwise specified.

## 4. Phase Space Integration

In order to compute the contributions to the moments, the analytic expressions of the  $\mathcal{O}(\alpha_s \Lambda_{QCD}^2/m_b^2)$  corrections given in Refs.[6, 7] need to be integrated over the phase space. This is most easily accomplished numerically and it is relatively straightforward. However, when a lower cut on the lepton energy is applied, or when the lepton energy is fixed to compute the spectrum, a few complications arise and it is worth describing the main steps we used in the integration.

Our starting point is the triple differential distribution:

$$\frac{d\Gamma}{d\hat{E}_\ell d\hat{q}^2 d\hat{u}} = \frac{G_F^2 m_b^5 |V_{cb}|^2}{16\pi^3} \theta[\hat{u}_+(\hat{q}^2) - \hat{u}] \times \\ \times \left\{ \hat{q}^2 W_1 - \left[ 2\hat{E}_\ell^2 - 2\hat{E}_\ell \hat{q}_0 + \frac{\hat{q}^2}{2} \right] W_2 \right. \\ \left. + \hat{q}^2 (2\hat{E}_\ell - \hat{q}_0) W_3 \right\}, \quad (6)$$

At the tree-level, only  $\hat{u} = 0$  is allowed, while gluon emission is restricted by

$$0 \leq \hat{u} \leq \hat{u}_+(\hat{q}^2) \equiv (1 - \sqrt{\hat{q}^2})^2 - \rho \\ 0 \leq \hat{q}^2 \leq (1 - \sqrt{\rho})^2. \quad (7)$$

To understand what happens when the lepton energy  $E_\ell$  is constrained to be larger than  $E_{cut}$

$$E_\ell \geq E_{cut} \equiv \frac{m_b}{2} \xi, \quad (8)$$

we first recall that without this cut the energy of the lepton would be in the range

$$\frac{q_0 - |\vec{q}|}{2} \leq E_\ell \leq \frac{q_0 + |\vec{q}|}{2} \quad (9)$$

where  $q_0$  and  $\vec{q}$  are the temporal and spatial components of the lepton pair four-momentum, which are uniquely determined for any value of  $\hat{q}^2$  and  $\hat{u}$ . We have two regions identified by

$$A: 0 \leq \hat{u} \leq \gamma(\hat{q}^2), \quad 0 \leq \hat{q}^2 \leq A_0 \equiv \frac{\xi(1-\rho-\xi)}{1-\xi}; \\ B: \begin{cases} \gamma(\hat{q}^2) \leq \hat{u} \leq u_+(\hat{q}^2), & \xi^2 \leq \hat{q}^2 \leq A_0 \\ 0 \leq \hat{u} \leq u_+(\hat{q}^2), & A_0 \leq \hat{q}^2 \leq (1 - \sqrt{\rho})^2 \end{cases}$$

where

$$\gamma(\hat{q}^2) = 1 + \hat{q}^2 \left( 1 - \frac{1}{\xi} \right) - \rho - \xi.$$

The integration over  $E_\ell$  can be performed analytically from the triple differential distribution and depends on the phase space region, as explained above. We are left with the integration over  $\hat{u}$  and  $\hat{q}^2$  in the various regions. The expressions we have to integrate can be split into three contributions: terms containing the Dirac delta  $\delta(\hat{u})$  and its derivatives  $\delta^{(n)}(\hat{u})$ , terms containing the plus distributions, and the finite part. Schematically,

$$\frac{d^2\Gamma_X}{du d\hat{q}^2} = \mathcal{D}_{i,X}(\hat{u}, \hat{q}^2) \delta^{(i)}[\hat{u}] \\ + \mathcal{U}_{i,X}(\hat{u}, \hat{q}^2) \left[ \frac{1}{\hat{u}^i} \right]_+ + \mathcal{T}_X(\hat{u}, \hat{q}^2) \theta(\hat{u}), \quad (10)$$

where we distinguish between  $X = full$ , for region  $B$ , and  $X = cut$  for region  $A$ . We next perform first the  $\hat{u}$  integration and reorganize the integral into three contributions in order to avoid spurious singularities at the boundary between regions  $A$  and  $B$ . In particular, we write the contribution of region  $B$  as the difference of two integrals starting from  $\hat{u} = 0$ . If  $\xi < (1 - \sqrt{\rho})$ , the integral over the phase space is the sum of

$$R_I = \int_0^{\xi^2} d\hat{q}^2 \int^{\gamma(\hat{q}^2)} d\hat{u} \frac{d^2\Gamma_{cut}}{d\hat{q}^2 d\hat{u}}, \\ R_{II} = \int_{\xi^2}^{A_0} d\hat{q}^2 \int^{\gamma(\hat{q}^2)} d\hat{u} \left[ \frac{d^2\Gamma_{cut}}{d\hat{q}^2 d\hat{u}} - \frac{d^2\Gamma_{full}}{d\hat{q}^2 d\hat{u}} \right], \\ R_{III} = \int_{\xi^2}^{(1-\sqrt{\rho})^2} d\hat{q}^2 \int^{u_+(\hat{q}^2)} d\hat{u} \frac{d^2\Gamma_{full}}{d\hat{q}^2 d\hat{u}}. \quad (11)$$

When  $\xi \geq (1 - \sqrt{\rho})$ , only one integral must be evaluated,

$$R_{IV} = \int_0^{A_0} d\hat{q}^2 \int^{\gamma(\hat{q}^2)} d\hat{u} \frac{d^2\Gamma_{cut}}{d\hat{q}^2 d\hat{u}}. \quad (12)$$

The integral of  $\delta^{(n)}(\hat{u})$  reduces to a one-dimensional integral over  $\hat{q}^2$  at  $\hat{u} = 0$ , but the presence of  $\theta[\hat{u}_{max}(\hat{q}^2) - \hat{u}]$  and  $\theta[\gamma(\hat{q}^2) - \hat{u}]$  from the  $\hat{u}$  cuts induces pinched contributions, *i.e.* terms containing  $\delta^{(n)}[\hat{u}_{max}(\hat{q}^2) - \hat{u}] \delta(\hat{u})$ . In the case of the integral of  $\delta^{(1)}(\hat{u})$  in  $R_{II}$ , for instance, we have

$$\int_{\xi^2}^{A_0} d\hat{q}^2 \int \theta[\gamma(\hat{q}^2) - \hat{u}] g(\hat{u}, \hat{q}^2) \delta^{(1)}(\hat{u}) d\hat{u} = \\ - \int_{\xi^2}^{A_0} \frac{\partial g}{\partial \hat{u}} \Big|_{\hat{u}=0} d\hat{q}^2 + \frac{\xi}{|1-\xi|} g(0, A_0). \quad (13)$$

A similar expression involving higher derivatives can be derived for  $\delta^{(2)}(\hat{u})$ . The resulting integrations can be easily performed in all regions.

For what concerns the plus distributions, it is convenient to use

$$f(\hat{u}) \left[ \frac{1}{\hat{u}} \right]_+ = \hat{u} f(\hat{u}) \left[ \frac{1}{\hat{u}^2} \right]_+ = \hat{u}^2 f(\hat{u}) \left[ \frac{1}{\hat{u}^3} \right]_+ \quad (14)$$

in order to have only one distribution with the maximal power, which in the case of the  $\lambda_1 \mathcal{O}(\alpha_s m_b^{-2})$  corrections is 3. The next step consists in applying the definition of plus distribution given in [6]

$$I = \int f(\hat{u}) \left[ \frac{1}{\hat{u}^3} \right]_+ d\hat{u} \\ \equiv \int_0^1 \frac{f(\hat{u}) - f(0) - \hat{u} f'(0) - \frac{1}{2} \hat{u}^2 f''(0)}{\hat{u}^3} d\hat{u}, \quad (15)$$

where  $f(\hat{u}) = \theta[\hat{u}_m(\hat{q}^2) - \hat{u}]g(\hat{u}, \hat{q}^2)$ , and  $\hat{u}_m(\hat{q}^2)$  depends on the region of integration. The differentiation of the  $\theta$  function leads to terms containing  $\theta[\hat{u}_m(\hat{q}^2)]$ ,  $\delta[\hat{u}_m(\hat{q}^2)]$  and  $\delta'[\hat{u}_m(\hat{q}^2)]$ . While  $R_I, R_{II}$  can now be numerically integrated without problems, in the case of  $R_{II}$  the  $\theta[\hat{u}_m(\hat{q}^2)]$  terms can be integrated over  $\hat{u}$  but present a logarithmic singularity in  $\hat{q}^2$ . The change of variable  $\hat{q}^2 \rightarrow \hat{q}^{2+} - (\hat{q}^{2+} - \hat{q}^{2-})\hat{u}$ , where  $\hat{q}^{2\pm}$  delimit the integration range in  $\hat{q}^2$ , moves the singularity to  $\hat{u} = 0$ , and make it possible to combine it with the singularity of the  $\delta[\hat{u}_m(\hat{q}^2)]$  and  $\delta'[\hat{u}_m(\hat{q}^2)]$  terms; the singularities cancel and their combination can be then be numerically integrated.

Explicitly,

$$I = \int_0^{\hat{u}_m(\hat{q}^2)} \frac{g(\hat{u}) - g(0) - \hat{u}g'(0) - \frac{1}{2}\hat{u}^2g''(0)}{\hat{u}^3} d\hat{u} - \int_{\hat{u}_m(\hat{q}^2)}^1 \frac{g(0) + \hat{u}g'(0) + \frac{1}{2}\hat{u}^2g''(0)}{\hat{u}^3} d\hat{u} + \int_0^1 \frac{(g(0) + g'(0)\hat{u})\delta[\hat{u}_m(\hat{q}^2)] - \frac{1}{2}g(0)\hat{u}\delta'[\hat{u}_m(\hat{q}^2)]}{\hat{u}^2} d\hat{u}. \quad (16)$$

where  $g$  is always a function of  $\hat{q}^2$  as well. Upon integration over  $\hat{q}^2$  the second line has a logarithmic singularity where the integration range of  $\hat{u}$  vanishes, in the lower right corner of region A. Instead, the third line has a logarithmic singularity for  $\hat{u} \rightarrow 0$ .

## 5. Preliminary Results

We first present the results of the numerical integration for each of the theoretical observables as a function of the lepton energy cut  $E_c$  in Figure 1. It is apparent that the new  $O(\alpha_s \Lambda_{\text{QCD}}^2/m_b^2)$  corrections are non-negligible, and compete with the NNLO contribution. This also drives the importance of continuing the calculation of the NLO corrections  $O(\alpha_s \Lambda_{\text{QCD}}^3/m_b^3)$ , as the leading order  $1/m_b^3$  corrections have large coefficients, and so one would similarly expect the NLO correction to also be competitive.

We perform the global fit following the same method as the previous determination [18], using for these proceedings only the case of [D] neglecting the external constraints on  $m_b$ . Provided as preliminary result we obtain Table 2, although we stress that further analysis is ongoing and will be presented shortly, and values are subject to minor adjustment. The right column of Table 2 is the results of the performed fit for the matrix element parameters, quark masses, and the

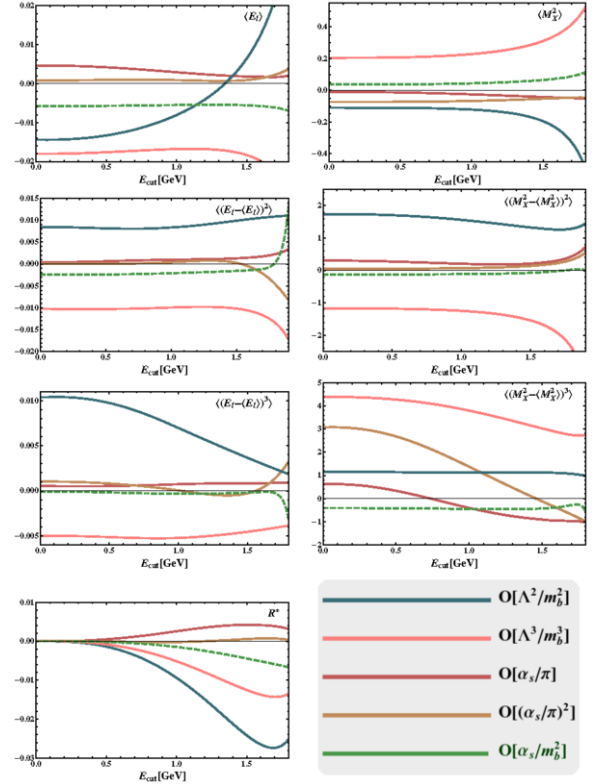


Figure 1: Theoretical observables  $\ell_{1,2,3}$ ,  $h_{1,2,3}$  and  $R^*$  as a function of  $E_{\text{cut}}$ .

resulting determination of  $V_{cb}$ . We find the new corrections lower  $V_{cb}$  by about 1%, leading to  $|V_{cb}|_{\text{prelim}} = (42.02 \pm 0.8) \times 10^{-3}$ .

$\bar{m}_c(3\text{GeV})$ [D]	$O(\alpha_s^2, m_b^{-2})$	$\delta$	$O(\alpha_s^2, \alpha_s m_b^{-2})$	$\delta$
$m_b^{\text{kin}}$	4.541	0.023	4.551	0.022
$m_c$	0.987	0.013	0.989	0.013
$\mu_\pi^2$	0.414	0.078	0.411	0.078
$\rho_D^2$	0.154	0.045	0.123	0.044
$\mu_G^2$	0.340	0.066	0.320	0.064
$\rho_{LS}^3$	-0.147	0.098	-0.141	0.097
$\text{BR}_{\text{c} \rightarrow \text{b}}(\%)$	10.65	0.16	10.65	0.16
$10^3  V_{cb} $	42.42	0.86	<b>42.02</b>	<b>0.80</b>

Table 2: Preliminary results using scenario [D] and mass constraint  $\bar{m}_c(3\text{GeV})$

Further discussion including the effects of the dependence of the strong coupling scale  $\alpha_s(\mu)$ , the renormalization scale of  $\mu_G(\mu_R)$ , and the effect of various parametrizations of the theoretical correlation of observables have been calculated and will be presented shortly.

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